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Covariant field equations for an electromagnetic string with mass points at the ends

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Abstract. A new set of covariant field equations for the interacting charge-monopole systems have been obtained which treats the string singularities of the electromagnetic potentials dynamically. The nonrelativistic monopole potentials are *derived* from the covariant theory.

1. Introduction

Several aspects of the otherwise remarkable and beautiful theory of magnetic monopoles (Dirac 1931, 1948) have been the subject of continued criticisms and controversies, in particular concerning the covariant form of the corresponding field theory (Usachev, 1973).

It becomes immediately clear that theory cannot be formulated as a *standard* field theory of the three interacting fields A_μ, Ψ and χ, representing the electromagnetic, the electric and the magnetic charge fields, respectively. The problem lies in the physical and mathematical significance of *singular* electromagnetic potentials which are not present in the usual field theories. This is a new feature distinguishing this theory from all other field theories we know.

The statement is often repeated that the charge quantisation condition makes the string singularities of the potential A_μ unobservable. But the story does not end here. For one thing, the charge quantisation depends essentially on the type of singularities assumed, and there is complete freedom in the choice of the singularities (Barut 1977, Barut and Schneider 1976). Then one has to deal with the vanishing of the electromagnetic current on the singularities in a dynamical and covariant way.

Dirac (1948) already in his classical field theory, fully aware of this problem, introduced, besides the potential $A_\mu(x)$, *additional dynamical variables* $y_\mu(\tau, \sigma)$, representing the coordinates of the extended singularities, for example stringlike. There is a manifestly covariant action which yields the electromagnetic field equations, the equations of motion of electric and magnetic charges, as well as the equations of motion for the singularity surfaces.

The purpose of this paper is to extend Dirac's action principle to its logical completion, without the complication of the so called 'Dirac veto' (§ 2), and to formulate the corresponding quantum field equations.

The equations for the singularity derived from the action principle tells us (Barut and Bornzin 1974, Balachandran 1976) that the electric charges respond to the singular potentials $A_\mu(x)$ in such a way that the normal component of the electric current vanishes on the singularity surface, as it should be, (Wentzel 1966). There are no other supplementary conditions, such as the Dirac veto. In this sense the singularities acquire a dynamical significance, although they are of electromagnetic nature, and their shape can be arbitrarily deformed by gauge transformation.

The new field equations, when massive electric charges and massive magnetic charges are both represented by Dirac fields, for example, show one-dimensional nonlocality corresponding to a integration along the string singularity. We also derive the explicit form of the singular potential which as a special case reduces to the non-covariant singular potentials (Dirac 1931, Schwinger 1966).

The firm interpretation of the theory has been used elsewhere (Barut 1978, to be published, reply to Usachev 1973) to answer various other problems which arose in connection with the derivation of charge quantisation condition, spin-statistics connections, etc.

We note finally that the problem of the Dirac veto has recently been discussed by Brandt and Primack (1977) in the context of the Wu–Yang formalism (1977) in dealing with singular potentials. In this formalism one uses different coordinate patches on different sides of the singularity string, as appropriate for manifolds which are not simply connected. However, one of the most important property of the charge-monopole system is the fact that such a system can have a total spin 1/2 even though both charges have spin 0. For this to happen it seems necessary to take into account fully the dynamical role of the string.

2. Classical and quantum field equations

By a covariant quantum field theory of the interacting electric and magnetic charges one means a rigorous and complete framework which *incorporates* the intuitive equations

$$F^{\mathrm{D}\ ,\nu}_{\mu\nu} = -j_\mu, \qquad \tilde{F}^{\mathrm{D}\ ,\nu}_{\mu\nu} = -k_\mu, \tag{1}$$

where the field $F^{\mathrm{Dirac}}_{\mu\nu}$ has only the point singularities corresponding to the electric current j_μ and the current k_μ of magnetic monopoles. In field theory we wish to represent j_μ and k_μ by, for example, if both poles are spinors,

$$\begin{aligned} j_\mu &= e\bar{\psi}\gamma_\mu\psi, \\ k_\mu &= g\bar{\chi}\gamma_\mu\chi. \end{aligned} \tag{2}$$

In equation (2), g is a pseudoscalar. (Some authors use $\chi\gamma_\mu\gamma_5\chi$, but this is not essential for covariance considerations.) The classical analogue of equations (2) are

$$\begin{aligned} j_\mu(x) &= \sum_i e_i \int \mathrm{d}z_\mu(s)\delta(x-z); \\ k_\mu(x) &= \sum_i g_i \int \mathrm{d}w_\mu(s)\delta(x-w), \end{aligned} \tag{3}$$

where $z(s)$ and $w(s)$ are the world lines of the poles.

A priori we wish of course to have a field theory in which *only* the fields ψ, χ and the electromagnetic field A_μ enters, with no other dynamical variables. The electromagnetic field in quantised theory must be described by a potential A_μ because of minimal coupling. We are interested in a completely covariant action principle and Lagrangian field equations and require a theory with vector potential $A_\mu(x)$, as opposed to a canonical theory, for example, upon which we shall comment later (§ 5).

It is here that the problem begins to differ from the usual quantum electrodynamics; the field satisfying equations (1) is *not* derivable from a potential. In the language of differential geometry the form F^{D} is not exact, i.e. not the differential of a form A. There is, however, a potential A_μ whose differential (i.e. curl) gives $F^{\mathrm{D}}_{\mu\nu}$ everywhere except along some singularity surface Λ:

$$F^{\mathrm{D}}_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} - \tilde{\Lambda}_{\mu\nu}, \tag{4}$$

where $\tilde{\Lambda}_{\mu\nu}$ (the dual of a tensor $\Lambda_{\mu\nu}$) is zero everywhere except on Λ. We have intentionally labelled the singularity surface and the singular field by the same letter. The precise form of Λ and $\tilde{\Lambda}_{\mu\nu}$ will be given later.

The potential A_μ naturally defines a new field $F^{\mathrm{M}}_{\mu\nu}$ by

$$A_{\nu,\mu} - A_{\mu,\nu} \equiv F^{\mathrm{M}}_{\mu\nu}. \tag{5}$$

Hence

$$F^{\mathrm{M}}_{\mu\nu} = F^{\mathrm{D}}_{\mu\nu} + \tilde{\Lambda}_{\mu\nu}. \tag{6}$$

It follows from equations (1), (4), and (6) immediately that

$$\begin{aligned} F^{\mathrm{M}}_{\mu\nu}{}^{,\nu} &= -j_\mu + \tilde{\Lambda}_{\mu\nu}{}^{,\nu} \equiv -j_\mu - j^{\mathrm{sing}}_\mu \\ \tilde{F}^{\mathrm{M}}_{\mu\nu}{}^{,\nu} &= 0, \end{aligned} \tag{7}$$

and

$$k_\mu = \Lambda_{\mu\nu}{}^{,\nu}. \tag{8}$$

Equations (7) show that our starting equations (1) are atuomatically embedded in ordinary Maxwell equations (7), hence the notation $F^{\mathrm{Maxwell}}_{\mu\nu}$. But this new maxwellian field has in addition to the electric current j_μ, the singular current $j^{\mathrm{singular}}_\mu$ which produces the singular field $\tilde{\Lambda}_{\mu\nu}$, introduced in (4).

The action S from which equations (1) and (7) *and* the equation of motion for the poles are derived is

$$S = \tfrac{1}{4}\int \mathrm{d}x\, F^{\mathrm{D}}_{\mu\nu} F^{\mathrm{D}\mu\nu} + \int \mathrm{d}x\, j_\mu(x) A^\mu(x) + S_{\mathrm{e}} + S_{\mathrm{g}}, \tag{9}$$

where S_{e} and S_{g} are the free actions of the electric and magnetic charges respectively. In view of (6), (9) can be rewritten as

$$\begin{aligned} S = \tfrac{1}{4}\int \mathrm{d}x\, F^{\mathrm{M}}_{\mu\nu} F^{\mathrm{M}\mu\nu} - \tfrac{1}{2}\int \mathrm{d}x\, \tilde{\Lambda}_{\mu\nu} F^{\mathrm{M}\mu\nu} + \tfrac{1}{4}\int \mathrm{d}x\, \tilde{\Lambda}_{\mu\nu}\tilde{\Lambda}^{\mu\nu} \\ + \int \mathrm{d}x\, j_\mu(x) A^\mu(x) + S_{\mathrm{e}} + S_{\mathrm{g}}. \end{aligned} \tag{10}$$

The second term, by (5) and partial integration, is equivalent to

$$-\tfrac{1}{2}\int \mathrm{d}x\, \tilde{\Lambda}_{\mu\nu} F^{\mathrm{M}\mu\nu} = -\int \mathrm{d}x\, \tilde{\Lambda}_{\mu\nu}{}^{,\nu} A^\mu = \int \mathrm{d}x\, j^{\mathrm{sing}}_\mu A^\mu. \tag{11}$$

Thus, the interaction of the singular current j^{sing}_μ with the field A^μ, although not appearing explicitly in (9), is contained therein. The variation of A^μ in S leads immediately to the Maxwell's equations (7). For the electric charges we get, in the classical case, from (3)

$$m_e \ddot{z}_\mu(s) = eF^{M,\nu}_{\mu\nu} \dot{z}_\nu(s), \tag{12}$$

and in quantum case from (2)

$$[\gamma^\mu(-\mathrm{i}\partial_\mu - eA_\mu) - m_e]\psi(x) = 0. \tag{13}$$

Both of these equations show that the charged particles respond also to the singular fields $\tilde{\Lambda}_{\mu\nu}$ contained in A_μ, or its differential $F^M_{\mu\nu}$.

We cannot vary the coordinates of magnetic poles and those of $\Lambda_{\mu\nu}$ independently, as they are coupled by equation (8). In fact (8) shows that k_μ is essentially the 'boundary' of the singular field $\Lambda_{\mu\nu}$. In order to see this and in order to perform a proper variation, we now choose a form for $\Lambda_{\mu\nu}$. All the equations up to now are independent of the choice of $\Lambda_{\mu\nu}$.

For a two-dimensional singularity surface Λ in the Minkowski space (i.e. world sheet of a string), we can write in the classical case

$$\Lambda_{\mu\nu}(x) = -g\int \mathrm{d}\tau\, \mathrm{d}\sigma\, (\dot{y}_\mu y'_\nu - \dot{y}_\nu y'_\mu)\delta(x-y), \tag{14}$$

where $y_\mu = y_\mu(\tau, \sigma)$ is the equation of the string in terms of the Lorentz invariant parameters τ and σ with $\dot{y} = \partial y/\partial\tau$ and $y' = \partial y/\partial\sigma$, i.e. $\Lambda_{\mu\nu}(x)$ is the differential 2-form (surface element) of Λ. In this case, we obtain, differentiating (14) and simplifying,

$$\Lambda_{\mu\nu}{}^{,\nu}(x) = g\int \mathrm{d}\tau\, \dot{y}_\mu(\sigma)\delta(x - y(\sigma))|^{\sigma_2}_{\sigma_1} = k^{(2)}_\mu(x) - k^{(1)}_\mu(x), \tag{15}$$

where σ_1 and σ_2 are the two endpoints of the string, hence k_μ is the magnetic current of the two oppositely charged monopoles, at the ends of the string. For a single monopole, we can let one of the endpoints move to infinity.

We then choose the equation of the string $y_\mu = y_\mu(\tau, \sigma)$ as the *third* independent field, and obtain from (10) with the help of (11), for each endpoint

$$w^{(i)}_\mu(\tau) \equiv y_\mu(\tau, \sigma = \sigma_i), \qquad i = 1, 2,$$

the equation

$$m_g \ddot{w}_\mu(\tau) = g\tilde{F}^D_{\mu\nu} \dot{w}^\nu(\tau), \tag{16}$$

and the equation

$$(\tilde{F}^D_{\mu\nu,\lambda} + \tilde{F}^D_{\nu\lambda,\mu} + \tilde{F}^D_{\lambda\mu,\nu})\dot{y}^\nu y'^\lambda = 0, \tag{17}$$

or

$$\epsilon_{\mu\nu\lambda\rho}\dot{y}^\nu y'^\lambda j^\rho(y) = 0. \tag{17$'$}$$

The first of these equations (16), gives the expected equation of motion of monopoles which respond to the field $\tilde{F}^D$ (and not F^M as was the case for the electric charges). The second, (17), states that the normal component of the electric current on the singularity surface must vanish, which is automatically satisfied because electric charges respond to the field F^M with singularities.

2.1. *Remark on the Dirac veto and gauge invariance*

From equation (17) Dirac concluded that

$$(\tilde{F}^{\mathrm{D}}_{\mu\nu,\lambda}+\tilde{F}^{\mathrm{D}}_{\nu\lambda,\mu}+\tilde{F}^{\mathrm{D}}_{\lambda\mu,\nu})|_{x=y}=0.$$

This is equivalent to

$$F^{\mathrm{D}\,,\nu}_{\mu\nu}|_{x=y}=j_\mu|_{x=y}=0,$$

which says that the electric current on the singularity sheet is zero, or 'a string must never pass through a charged particle'. This is the origin of, what has been called subsequently, the Dirac veto. If this were the case, the string would be completely unphysical, and this Dirac wanted to achieve. This requirement is an extra constraint, 'not derivable from the action', hence not acceptable. We see however that only the weaker condition (17) or (17′) follows from the action principle, and this weaker condition is always satisfied.

Because the current j_μ is gauge invariant but the string is deformable, one might ask whether (17′) violates gauge invariance. Since we have introduced the string functions $y_\mu(\sigma,\tau)$ as dynamical variables, and since y_μ enters in the singular potential A_μ or $F^{\mathrm{M}}_{\mu\nu}$, it follows from equations (12) or (13) that the solution $\dot{z}_\nu$, or $\psi(x)$, of these equations will depend implicitly on the choice of y_μ. But condition (17′) is true for every choice of y_μ. We have thus, *in addition* to the usual gauge invariance $A_\mu \to A_\mu + f_{,\mu}$ under which j_μ is of course *gauge invariant*, the additional transformations of changing the coordinates of the string (or reparametrisation of the string). The action principle, hence all the equations that follow from it, admit these transformations, in particular equation (17′).

2.2. *Derivation of wave equations*

It remains only to derive the quantum version of (16). The quantisation of the string variables $y_\mu(\tau,\sigma)$ is a new problem and will depend on further physical considerations concerning the significance of the string. In the action (9), we have introduced mass terms at the endpoints only. It would be equally possible to introduce a mass distribution along the string.

For the case where we wish to quantise the motion of the endpoints only—and thus to establish contact with the previous work—we proceed as follows:

We choose a coordinate condition such that

$$y^0(\tau,\sigma)=t, \tag{18}$$

and then write

$$y^\mu(\tau,\sigma)=w^\mu(t)+u^\mu(\sigma), \tag{19}$$

where $u^\mu(\sigma)$ is a spacelike four vector, $u^\mu(\sigma_1)=0$ and $u^\mu(\sigma_2)$ being associated with the two endpoints. Now equation (14) becomes

$$\begin{aligned}\Lambda_{\mu\nu}(x)&=-g\int \mathrm{d}\tau\,\dot{w}_\mu\int_{\sigma_1}^{\sigma_2}\mathrm{d}\sigma\,u'_\sigma\delta(x-w-u)-(\mu\leftrightarrow\nu)\\&=-g\int \delta(x-w-u)\,\mathrm{d}w_\mu\wedge\mathrm{d}u_\nu,\end{aligned} \tag{20}$$

or, with (3) and 2,

$$\begin{aligned}\Lambda_{\mu\nu}(x) &= -\int k_\mu(x-u)\wedge \mathrm{d}u_\nu \\ &= -g\int_{\sigma_1}^{\sigma_2} \bar{\chi}(x-u)\gamma_\mu\chi(x-u)\frac{\mathrm{d}u_\nu}{\mathrm{d}\sigma}\mathrm{d}\sigma - (\mu\leftrightarrow\nu),\end{aligned} \tag{21}$$

and represents an integral of the magnetic current along the string. The differential of (21) gives again, as in (15), the magnetic currents at the endpoints:

$$\begin{aligned}\Lambda_{\mu\nu}{}^{,\nu}(x) &= -\int \partial^\nu k_\mu(x-u)\wedge \mathrm{d}u_\nu \\ &= +\int \frac{\partial k_\mu(x-u)}{\partial u_\nu}\wedge \mathrm{d}u_\nu = \int_{\sigma_1}^{\sigma_2} \mathrm{d}\, k_\mu(x-u(\sigma)) \\ &= k_\mu(x-u(\sigma))\big|_{\sigma_1}^{\sigma_2}.\end{aligned} \tag{22}$$

Hence

$$\tilde{F}^{\mathrm{D}\ ,\nu}_{\mu\nu} = -\Lambda_{\mu\nu}{}^{,\nu} = -k_\mu(x-u(\sigma))\big|_{\sigma_1}^{\sigma_2}. \tag{23}$$

Note: The quantisation is performed in equation (21) by replacing k_μ with $\chi\gamma_\mu\chi$ in analogy to the electric case. More precisely, to one end of the string (with mass) we associate a wavefunction; the wavefunction of the rest of the string is then indicated by the argument in $\chi(x-u)$. It would also be possible to associate a wave *functional* to the whole string (cf § 4).

The action terms containing the singularity including the endpoints are, according to (10) and (11):

$$-\tfrac{1}{2}\int \mathrm{d}x\,\Lambda_{\mu\nu}\tilde{F}^{\mathrm{M}\mu\nu} + \tfrac{1}{4}\int \mathrm{d}x\,\tilde{\Lambda}_{\mu\nu}\tilde{\Lambda}^{\mu\nu} + S_{\mathrm{g}}.$$

For the case that S_{g} is a Dirac Lagrangian for a field χ with mass m_{g}, we obtain using (21) and varying with respect to $\bar{\chi}$ the equation:

$$(\gamma^\mu p_\mu - m_{\mathrm{g}})\chi(x) = g\int_{\sigma_1}^{\sigma_2} \tilde{F}^{\mathrm{D}}_{\mu\nu}(x)\chi(x-u)[\gamma^\mu\,\mathrm{d}u^\nu - \gamma^\nu\,\mathrm{d}u^\mu]. \tag{24}$$

Collecting all the equations together, our final coupled equations for fields A_μ, ψ and χ, with the usual gauge condition

$$A^\mu{}_{,\mu} = 0, \tag{25}$$

are

$$\begin{aligned}&[\gamma^\mu(-\mathrm{i}\partial_\mu - eA_\mu(x)) - m_{\mathrm{e}}]\psi(x) = 0 \\ &[\gamma^\mu(-\mathrm{i}\partial_\mu) - m_{\mathrm{g}}]\chi(x) = g\int_{\sigma_1}^{\sigma_2} \tilde{F}^{\mathrm{D}}_{\mu\nu}(x)\chi(x-u)\gamma^\mu\wedge\mathrm{d}u^\nu \\ &\Box A_\mu(x) = e\bar{\psi}\gamma_\mu\psi - g\epsilon_{\mu\nu\lambda\rho}\int_{\sigma_1}^{\sigma_2}\frac{\partial}{\partial u_\nu}[\bar{\chi}(x-u)\gamma^\lambda\chi(x-u)]\,\mathrm{d}u^\rho.\end{aligned} \tag{26}$$

All three equations involve the singularity and we have a one-dimensional non-locality, an integration along the string.

Finally we give a solution of the third equation in (26) for the potential:

$$A_\mu(x) = e\int \mathrm{d}x'\bar{\psi}(x')\gamma_\mu\psi(x')D(x-x') + g\int \mathrm{d}x'\,\epsilon_{\mu\nu\lambda\rho}k^\lambda(x'-u)\wedge \mathrm{d}u^\rho \frac{\partial}{\partial x_\nu}D(x-x'). \tag{27}$$

It is instructive to evaluate (27) for a special choice of the string. If we set the space-like vector as

$$u_\mu = (0, (\pi-\sigma)R\hat{\boldsymbol{n}}), \qquad u'_\mu = (0, -R\hat{\boldsymbol{n}}), \tag{28}$$

where $\hat{\boldsymbol{n}}$ is a unit vector, σ a parameter and R a distance, we have for the space components of the singular part of potential A_μ, from (27),

$$\begin{aligned} A_i(x) &= g\epsilon_{ijk}\int \mathrm{d}x'\,k^0(x'-u)R\hat{n}^j\frac{\partial}{\partial x_k}D(x-x') \\ &= g\epsilon_{ijk}\int \mathrm{d}x''\,k^0(x'')\hat{n}^j\int \mathrm{d}\sigma'\frac{\partial}{\partial x_k}D(x-u-x''), \qquad \sigma' = (\pi-\sigma)R \\ &= g\epsilon_{ijk}\hat{n}^j\int \mathrm{d}x''\,k^0(x'')\int \mathrm{d}\sigma'\frac{\partial}{\partial r_k}\frac{\delta(x_0-x_0''-|\boldsymbol{x}-\boldsymbol{x}''-\sigma'\hat{\boldsymbol{n}}|)}{|\boldsymbol{x}-\boldsymbol{x}''-\sigma'\hat{\boldsymbol{n}}|}, \end{aligned}$$

which for $R \gg r$ simplifies to

$$A_i(x) = g\int \mathrm{d}^3x''\,k^0(x'')D_i(\boldsymbol{x}-\boldsymbol{x}'', \hat{\boldsymbol{n}}) \tag{29}$$

here

$$D(\boldsymbol{r}, \hat{\boldsymbol{n}}) = \frac{\boldsymbol{r}\times\hat{\boldsymbol{n}}}{r(r-(\boldsymbol{r}\cdot\hat{\boldsymbol{n}}))}, \tag{30}$$

for a single string and similarly,

$$D(\boldsymbol{r}, \hat{\boldsymbol{n}}^{(i)}) = \frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{r}\times\hat{\boldsymbol{n}}^{(i)}}{r(r-(\boldsymbol{r}\cdot\hat{\boldsymbol{n}}^{(i)}))} \tag{30'}$$

for N string. Equation (29) with (30') for $N=2$, $\hat{\boldsymbol{n}}^{(1)} = -\hat{\boldsymbol{n}}^{(2)}$, is precisely the potential introduced by Schwinger (1966) which we now see is a special case of the covariant formula (27).

3. Covariance and infinite-component wave equations

A theory tells us what are the observables in that theory. In our case the action principle gives the following physical picture: We have electric charges interacting with pairs of magnetic monopoles which are at the endpoints of a magnetic flux line. For a single monopole the opposite pole may be thought to be at infinity, hence the flux line goes to infinity. The flux line is real although it has no inertia except the monopole masses at its endpoints and its shape can be chosen arbitrarily. It enters into the field theory through the space-like vector $u^\mu(\sigma)$ which must be treated as an additional dynamical variable, and must be transformed as a four vector under Lorentz transformations. The generators of the Lorentz transformations $J_{\mu\nu}$ depend on $u^\mu(\sigma)$ as well. In other words we have new realisations on $J_{\mu\nu}$ in a function space whose elements vanish along the singularity surfaces, or equivalently in spaces with new topologies, namely spaces cut along the singularity surfaces. ($J_{\mu\nu}$ as differential operators are singular because they contain singular potentials, hence they must act on functions which vanish at these singularities.)

It is not unusual that a theory contains new 'gauge coordinates' of the type $u^\mu(\sigma)$. The Dirac theory of the spinning electron contains the vector γ_μ which must transform like a vector but whose choice is arbitrary. Here the components of γ_μ are represented by finite-dimensional matrices, in our case by functions over a real line segment σ, i.e. by infinite-dimensional matrices. In the infinite-dimensional Majorana equation, for example, Γ_μ's are infinite-dimensional matrices. In both cases the new coordinates represent new internal degrees of freedom (spin or other internal degrees of freedom). I have discussed elsewhere in detail the relation of the string coordinates to spin (Barut 1974).

In fact our equations (25) can be written as local infinite-component wave equations. Introducing an operator

$$K^\rho f(x) = \int_{\sigma_1}^{\sigma_2} f(x-u)u^\rho \,\mathrm{d}\sigma = \int f(x-u)\,\mathrm{d}u^\rho \tag{31}$$

we can rewrite equations (26) as

$$\begin{aligned}
&[\gamma^\mu(-i\partial_\mu - eA_\mu) - m_e]\psi = 0 \\
&[\gamma^\mu(-i\partial_\mu) - g\tilde{F}^{\mathrm{D}}_{\mu\nu}K^\nu\gamma^\mu - m_g]\chi = 0 \\
&\Box A_\mu = e\bar{\psi}\gamma_\mu\psi + g\epsilon_{\mu\nu\lambda\rho}\frac{\partial}{\partial x_\nu}K^\rho(\bar{\chi}\gamma^\lambda\chi)
\end{aligned} \tag{32}$$

Note that χ now has a spinor index α and an infinite-component index A on which the operators K^ρ act: $\chi_{\alpha A}(x)$.

Equations (32) are the set of field equations to be solved by Green function propagator techniques, or to be second quantised further by imposing commutation relations. The fields ψ and χ enter in an assymmetric way due to the presence of singular strings. Of course we can equally well attach the string to the electric charges; the duality still holds.

4. Generalisations

We have assumed mass points at the endpoints of the strings and treated them as spinors in order to establish contact with other field theories. However, although charged particles are correctly described by Dirac spinors ψ, the magnetic singularities are new objects and their proper quantisation is still an open subject. We might for example put bosons at the endpoints, because a spin 1/2 state will emerge anyway for the charge–monopole system even if the monopole is a boson. Or, we may not put any mass points at all at the endpoints, or we may assume a mass distribution along the string. All these cases, and others, are possible and have to be considered.

5. Conclusions

We have developed the Lagrangian formalism of the charge–monopole field theory and obtained covariant field equations which involve necessarily integrations over the singularity surfaces, in particular integrations along the string. The position of the singularity is immaterial given the endpoints. There seems to be no way of avoiding entirely the singularities which in the Dirac action principle must acquire a physical reality, as discussed in § 3. Even in theories based on canonical formalism the singularity enters into the theory at the end in the same way (Villarroel 1976).

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